

**what empirical data about measuring multifractals in the files in this Space**

Empirical data and methods for estimating multifractal properties in the files in this Space focus primarily on daily NASDAQ 100 returns from 1998–2015, as well as broader financial series and derivatives. The main approach is a **three-parameter model** that separately identifies structural tail risk, volatility clustering, and long-range dependence, fully falsifiable with robust diagnostics and simulation[[1]](#fn1)[[2]](#fn2). Below is a summary of sources, sample characteristics, estimation methods, baseline results, and checks.

**Sample and Data Sources**

* **Core sample:** Daily close-to-close log-returns of the NASDAQ 100, 1998–2015 (about 4,600 observations)[[1]](#fn1)[[2]](#fn2).
* **Asset universe extension:** Framework applicable to S&P 500, FX, commodities, and derivatives, calibrating three parameters across instruments[[3]](#fn3)[[4]](#fn4)[[5]](#fn5).

**Parameter Estimation and Transformations**

* **Tail index (α):** Quantifies heavy tails (“Noah effect”); estimated from empirical upper/lower tail slopes using Hill’s estimator or peaks-over-threshold, with stability across thresholds as a diagnostic.
* **Hurst exponent (H):** Measures persistence/anti-persistence in aggregated returns; estimated as slope of the log–log plot of the first-order structure function S₁ versus lag over 2–40 days.
* **Intermittency (λ):** Captures volatility clustering (“Moses effect”); estimated from the slope of the log-covariance of log-absolute returns with lag.
* **Scaling spectrum (ζ(q)):** Empirically regressed for q = [0.5, 1, 1.5, 2] to ensure empirical q matches the theoretical multifractal parabola.
* **Data integrity:** Adjusted closing prices used to compute stationary returns; scaling/robustness checks avoid microstructure and non-stationarity by focusing on intermediate lags[[1]](#fn1)[[2]](#fn2).

**Baseline Specification**

|  |  |
| --- | --- |
| Parameter | Full Sample (1998–2015) |
| Tail index α | 1.85 |
| Hurst exponent H | 0.53 |
| Intermittency λ | 0.22 |
| N (obs) | ≈4,600 |
| Std. Error | Robust, blockwise/overlapping for each regression[[1]](#fn1)[[2]](#fn2) |

* **Clustering:** λ rises in crisis/high-volatility eras, falls in calm—validated by sub-periods (dot-com boom, GFC, etc.).
* **Persistence:** H modestly above 0.5, revealing weak but persistent memory not present in pure random walks.
* **Tail risk:** α is stable, suggesting heavy-tailedness is a structural, not cyclical, feature.

**Robustness and Model Validation**

* **Simulation:** Synthetic series are generated by simulating log-volatility processes with the estimated λ, using innovations from stable or Pareto distributions (matching α), and validating the resulting simulated return series against the observed data for tails, q-scaling, and autocorrelation.
* **Diagnostics:**
  + Tail slope and scaling curve fit compared (empirical vs simulated, MAPE and KS distance).
  + Volatility-covariance error (slope and R²).
  + Exceedance probabilities at high quantiles (drift, tails).
  + Regime/stability checks: rolling or regime-specific estimation, MaxEnt regime detection.
  + Reproducibility: fixed RNG seeds, code blocks, and recorded parameter sets.
* **Cluster regime detection:** MaxEnt-based regime shifts inferred from observable parameter changes, with further tests on martingale property and Kelly-optimality[[2]](#fn2).
* **Finite-sample bias check:** Overlapping blocks and threshold-stability for tail estimation, scaling bands sensitivity (3–30, 5–50 day lags).
* **Clustering and scaling cross-validation:** H and λ robust to innovation distribution choice in the MRW/MF framework[[1]](#fn1)[[2]](#fn2).

**Threats to Identification and Data Limitations**

* **Possible threats:**
  + Microstructure bias at low lags.
  + Non-stationarity at very long lags.
  + Model misspecification: assumes three-parameter sufficiency, which is empirically validated by simulation, but additional parameters may be needed for higher-order moments.
  + Single-asset focus—multivariate scaling less documented in current files, but flagged for future work.
* **Statistical errors:** Standard errors clustered overlap blocks to account for dependency; convergence and stability checked via sub-sample estimates.

**Main Table: NASDAQ 100 Multifractal Parameters**

|  |  |  |  |
| --- | --- | --- | --- |
| Period | α (Tail) | H (Persistence) | λ (Clustering) |
| 1998–2003 | — | 0.55 | 0.25 |
| 2004–2006 | — | 0.51 | 0.14 |
| 2007–2015 | — | 0.54 | 0.24 |
| Full sample | 1.85 | 0.53 | 0.22 |

* Sub-period swings in λ correspond to market stress, supporting empirical interpretation of clustering.
* α stable, H slightly >0.5 in all samples—structural persistence, not regime driven[[1]](#fn1)[[2]](#fn2).

**Two Key Robustness Checks**

1. **Parameter stability:** Estimates robust to choice of scaling window (lag band) and innovation distribution in simulation.
2. **Simulation validation:** All main empirical stylized facts—heavy tails, volatility clustering, scaling—matched in synthetic series using only empirically estimated parameters.

**Summary of Empirical Approach**

* **Sample:** NASDAQ 100 daily returns, 1998–2015, ~4,600 obs. Adjusted prices, focus on intermediate lags, robust diagostics.
* **Empirical strategy:** Estimate α (Hill or POT), H (structure function slope), λ (covariance slope); validate with simulation and rolling regimes; cluster standard errors; check model fit by moments, tails, and scaling.
* **Baseline results:** α ≈ 1.85, H ≈ 0.53, λ ≈ 0.22; λ varies with regimes; all stylized facts reproduced.
* **Robustness:** Alternative scaling windows, innovation laws, MaxEnt regimes, simulation length, threshold checks, blockwise SEs.
* **Threats:** Microstructure, non-stationarity, model sufficiency. Multivariate extension and spectral clustering left for future work[[1]](#fn1)[[2]](#fn2)[[3]](#fn3)[[4]](#fn4).

**For reference:** Further files in the Space (notably the comprehensive MAPM and JFIN-Advanced-Methods documents) extend this methodology to other asset classes, derivative payoffs, and discuss implications for derivative pricing, Kelly criterion enforcement, and spectral diagnostics of market noise, all with clear empirical recipes grounded in three core parameters[[3]](#fn3)[[5]](#fn5)[[4]](#fn4).

If additional quantitative breakdowns or code for reproducibility are required, these can be provided per standard log/recording conventions[[1]](#fn1)[[2]](#fn2).

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1. multifractals.docx

1. ChatGPT-Article-with-Simulations-v0.2.docx

1. Multi-fractal-full-thread-v0.1-2026.09.02.docx

1. MAPM-articlev\_1-2025.09.02.docx

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